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Construction of a series of balanced quaternary designs based on BIB designs and its applications in intercropping experiments

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ABSTRACT

This paper is concerned with the recursive construction of balanced quaternary (BQ) designs through a set of balanced incomplete block (BIB) designs. An illustrative example in each case has been added separately. The efficiency of BQD has also been computed in addition to their applications to the intercropping experiment(maize and cowpea).

Keywords: Balanced incomplete block design; balanced quaternary design, incidence matrix, intercropping experiments

Balanced *n*-ary designs were introduced by Tocher (1952) as a generalization of balanced incomplete block design. In this design, the incidence matrix can take any value of *n* out of possible values, often 0,1,2….(n-1). If n = 3,we get a ternary block design. Bilington (1984,1989) have extensively given results on balanced ternary designs. These designs may not exist for all parametric combinations or even if exist may require a large number of replications. In the present paper, we provide a new method of construction of BQD using a set of balanced incomplete block designs.

Balanced Quaternary Design (BQD)

A balanced n-ary design is a collection of B multisets each of size K, chosen from a set of size V in such a way that each of the V elements occurs R times altogether and 0,1,2,……or n-1 times in each block and each pair of distinct elements occurs Λ times. So the inner product of any two distinct rows of the V × B incidence matrix of the balanced n-ary design is Λ. A balanced n-ary design where n = 4 is known as the balanced quaternary design which is a collection of B blocks, each of cardinality K(K ≤ V), chosen from a set of size V in such a way that each of the V treatments occurs R times altogether, each of the treatments occurring once in a precisely Q_1 blocks, twice in precisely Q_2 blocks and thrice in precisely Q_3 blocks, with incidence matrix having inner product of any two rows Λ is denoted by balanced quaternary design (BQD) having parameters (V, B, Q_1, Q_2, Q_3, R, K, Λ). It is to be noted that $Q_1 + 2Q_2 + 3Q_3 = R$ (Gupta *et al.* 1995; Sarvate and Seberry 1993).

Construction

Theorem 2.1 The existence of a BIB design with parameters $V = 2k + 1$, b, r, k, λ with $b = 3r - 2\lambda$ implies the existence of balanced quaternary design (BQD) with following parameters V = 2(k+1),

$$B = \frac{b(2b-1)(2b-2)}{2}, \quad Q_1 = \frac{b^2(b-1)}{2},$$

$$Q_2 = \frac{b^2(b-1)}{2}, \quad Q_3 = \frac{b(b-1)(b-2)}{6},$$

$R = b(2b^2 - 3b + 1)$, $K = 3(k+1)$, $\Lambda = (b-1)[r(2b-3) + 2b^2]$

Proof : With the existing BIB design, a self complementary BIB design with the parameters V = 2(k+1), $b' = 2b$, $r' = b$, $k' = k+1$, $\lambda' = r$ can be constructed (Mitra and Mandal 1998). Then BQD are constructed by taking the combination of three blocks of the self complementary design together at a time.

$$\text{Hence} = \binom{2b}{3} = \frac{b(2b-1)(2b-2)}{3}$$

Therefore, the total number of blocks is

$$B = \frac{b(2b-1)(2b-2)}{3}$$

The parameters *V, B*, and *K* need no explanation.

Remaining parameters are explained below :

Q_1 : Let us consider a block containing a particular treatment *x*. This will occur by taking the combination of *r* and *b-r* which is equal to

$$Q_1 = \binom{b}{1} \times \binom{b}{2} = \frac{b^2(b-1)}{2}$$

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Q_2 : This will occur by taking the combination of two treatments i.e. *r* and *b-r* which is equal to

$$Q_2 = \binom{b}{2} \times \binom{b}{1} = \frac{b^2(b-1)}{2}$$

$$Q_3 = \binom{b}{3} = \frac{b(b-1)(b-2)}{6}$$

R : Replication number *R* for treatment *x* is $R = Q_1 + 2Q_2 + 3Q_3$.

Hence, $R = b(2b^2$ □ $3b+1)$.

K : 3(*k*+1)

Λ : This parameter will consist of (3,3), (3,2), (2,3), (3,1), (1,3), (2,2), (2,1), (1,2), (1,1) ordered pairs of treatments.

For ordered pair (3,3), we consider 3's of the total λ's. Therefore , it is equal to $\binom{r}{3}$.

For ordered pair (3,2) and (2,3), we consider the 2's of λ's and one of (r - λ) which is equal to $\binom{r}{2} \times \binom{b-r}{1}$.

For ordered pair (3,1) and (1,3), we consider the 1's of λ's and 2's of (r - λ) which is equal to $\binom{r}{1} \times \binom{b-r}{2}$

For ordered pair (2,2),we consider 2's of λ's and one of (0,0) and combination of (1,1,0) and (0,1,1) , we consider one of λ's and 2's of (r-λ) which is equal to

$$\binom{r}{2}\binom{r}{1} + \binom{r}{1}\binom{b-r}{1}\binom{b-r}{1}$$

For ordered pair (2,1) and (1,2), we consider the one of λ's and one of the (r - λ) and (b – 2r + λ) and other combination of (1,0,0) and (0,1,0), we consider 2's and one of (r – λ) which is equal to

$$\binom{r}{1}\binom{b-r}{1}\binom{r}{1} + \binom{b-r}{2}\binom{b-r}{1}$$

For ordered pair (1,1), we consider one of λ's and 2's of (b – 2r + λ) and other combination of (1,0,0) and (0,1,0), we consider of 2's of (r - λ) and one of (b - 2r + λ) which is equal to

$$\binom{r}{1}\binom{r}{2} + \binom{r}{1}\binom{b-r}{1}\binom{b-r}{1}$$

Thus,

$$\Lambda = 9\binom{r}{3} + 12\binom{r}{2} \times \binom{b-r}{1} + 6\binom{r}{1} \times \binom{b-r}{2} +$$

$$4\left[\binom{r}{2}\binom{r}{1} + \binom{r}{1}\binom{b-r}{1}\binom{b-r}{1}\right] +$$

$$4\left[\binom{r}{1}\binom{b-r}{1}\binom{r}{1} + \binom{b-r}{2}\binom{b-r}{1}\right] +$$

$$\binom{r}{1}\binom{r}{2} + \binom{r}{1}\binom{b-r}{1}\binom{b-r}{1}$$

$\Lambda = (b-1)[r(2b-3) + 2b^2]$

Hence Q.E.D.

Corollary 2.1 : The existence of a BIB design with parameters V = b= 4t-1, r = k = 2t-1, λ = t-1,implies the existence of a BQD with parameters, V = 4t, $B = \frac{(4t-1)(8t-3)(8t-4)}{3}$, $Q_1 = (2t-1)(4t-1)^2$, $Q_2 = (2t-1)(4t-1)^2$, $Q_3 = \frac{(4t-1)(4t-2)(4t-3)}{6}$, $R = (4t-1)(32t^2 - 28t + 6)$, K = 6t, $\Lambda = (4t-2)[(2t-1)(8t-5) + 2(4t-1)^2]$.

Corollary 2.2: The existence of a BIB design with parameters V = 2t-1,b = 4t-2, r = 2t-2, k = t-1, λ = t-2, implies the existence of a BQD with parameters,V = 2t, $B = \frac{(4t-2)(8t-5)(8t-6)}{3}$, $Q_1 = \frac{(4t-2)^2(4t-3)}{2}$, $Q_2 = \frac{(4t-2)^2(4t-3)}{2}$, $Q_3 = \frac{(4t-2)(4t-3)(4t-4)}{6}$, $R = (4t-2)(32t^2 - 44t + 15)$, K = 2t, $\Lambda = 2(4t-3)[(t-1)(8t-7) + (4t-2)^2]$.

Corollary 2.3 : The existence of a BIB design with parameters V = 2t-1, b = 4t-2, r = 2t-2, k = t-1, λ = t-2, implies the existence of a BQD with parameters, V = 2t, B = (2t-1)(4t-3), Q_1 = 4t(t-1), Q_2 = (t-1)(2t-3), K = 2(t-1), $R = 2(4t^2-7t+3)$, $\Lambda = 8t^2-20t+12$.

Example 2.1: Let us consider BIB design with parameters V=b=3, r=k=1, λ=0. It implies self complementary BIB design with parameters V = 4, $b' = 6,\ r' = 3,\ k' = 2,\ \lambda' = 1$. On applying Theorem 2.1, it is developed as BQD (Table 1).

Table1: The number of blocks in BQD with parameters V=4, B=20, Q_1=9, Q_2=9, Q_3=1, R=30, K=6, Λ= 42

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}	B_{19}	B_{20}
1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1
2	2	1	1	2	1	1	1	1	1	2	2	2	2	2	1	2	2	1	1
3	2	2	2	3	3	2	2	2	1	3	3	2	2	2	2	3	2	2	2
4	3	3	2	3	3	3	3	2	2	3	3	3	3	2	2	3	3	3	2
4	4	4	4	4	4	4	3	3	3	4	4	4	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	3

Note: Efficiency of this design is-0.933

Theorem 2.2: The existence of a BIB design with parameters V = 2k + 1, b, r, k, λ with b = 3r - 2λ implies the existence of balanced quaternary design (BQD) with following parameters V = 2(k + 1), B = $\frac{b(2b-1)(2b-2)}{3}-2b$, Q_1 = $\frac{b^2(b-1)}{2}-b$, Q_2 = $\frac{b^2(b-1)}{2}-b$, Q_3 = $\frac{b(b-1)(b-2)}{6}$, R = b(2b²-3b-2), K = 3(k+1), Λ = (b-1)[r(2b-3) + 2b²] – (4b+r).

Proof: With the existing BIB design, a self complementary BIB design with the parameters V = 2(k+1), $b'=2b,\ r'=b,\ k'=k+1,\ \lambda'=r$ can be constructed (Mitra and Mandal 1998). Then BQD are constructed by taking the combination of two blocks of the self complementary design together at a time.

Hence, $\binom{2b}{3}-2b=\frac{b(2b-1)(2b-2)}{3}-2b$.

Therefore, the total number of blocks is

B = $\frac{b(2b-1)(2b-2)}{3}-2b$.

The parameters *V*, *B*, and *K* need no explanation.

Remaining parameters are explained below:

Q_1 : Let us consider a block containing a particular treatment *x*. This will occur by taking the combination of *r* and *b-r* which is equal to

$$Q_1=\binom{b}{1}\times\binom{b}{2}-b=\frac{b^2(b-1)}{2}-b$$

Q_2 : This will occur by taking the combination ot two treatments i.e. *r* and *b-r* which is equal to

$$Q_2=\binom{b}{1}\times\binom{b}{2}-b=\frac{b^2(b-1)}{2}-b$$

$$Q_3=\binom{b}{3}=\frac{b(b-1)(b-2)}{5}$$

R : Replication number *R* for treatment *x* is *R* = Q_1 + $2Q_2$ + $3Q_3$.

Hence, R = b(2b²-3b-2).

K : 3(k+1)

Λ : This parameter will consist of (3,3), (3,2), (2,3), (3,1), (1,3), (2,2), (2,1), (1,2), (1,1) ordered pairs of treatments.

For ordered pair (3,3), we consider 3's of the total λ's. Therefore , it is equal to $\binom{r}{3}$.

For ordered pair (3,2) and (2,3), we consider the 2's of λ's and one of (r-λ) which is equal to $\binom{r}{2}\times\binom{b-r}{1}$.

For ordered pair (3,1) and (1,3), we consider the 1's of λ's and 2's of (r-λ) which is equal to $\binom{r}{1}\times\binom{b-r}{2}$.

For ordered pair (2,2),we consider 2's of λ's and one of (0,0) and combination of (1,1,0) and (0,1,1) , we consider one of λ's and 2's of (r-λ) which is equal to

$\binom{r}{2}\binom{r}{1}+\binom{r}{1}\binom{b-r}{1}\binom{b-r}{1}$.

For ordered pair (2,1) and (1,2), we consider the one of λ's and one of the (r-λ) and (b-2r+λ) and other combination of (1,0,0) and (0,1,0), we consider 2's and one of (r-λ) which is equal to

$$\binom{r}{1}\binom{b-r}{1}\binom{r}{1}+\binom{b-r}{2}\binom{b-r}{1}$$

For ordered pair (1,1), we consider one of λ's and 2's of (b-2r+λ) and other combination of (1,0,0) and (0,1,0), we consider of 2's of (r-λ) and one of (b-2r+λ) which is equal to

$$\binom{r}{1}\binom{r}{2}+\binom{r}{1}\binom{b-r}{1}\binom{b-r}{1}$$

Thus,

$$\Lambda=9\binom{r}{3}+12\binom{r}{2}\times\binom{b-r}{1}+6\binom{r}{1}\times\binom{b-r}{2}+$$

$$4\binom{r}{2}\binom{r}{1}+\binom{r}{1}\binom{b-r}{1}\binom{b-r}{1}+$$

$$4\binom{r}{1}\binom{b-r}{1}\binom{r}{1}+\binom{b-r}{2}\binom{b-r}{1}+$$

$$\binom{r}{1}\binom{r}{2}+\binom{r}{1}\binom{b-r}{1}\binom{b-r}{1} - 4r - 4(b - r) - r.$$

$\Lambda = (b-1)[r(2b-3) + 2b^2] - (4b + r)$.

Hence Q.E.D.

Corollary 2.4: The existence of a BIB design with parameters V = b = 4t-1, r = k = 2t - 1, λ = t - 1, implies the existence of a BQD with parameters, V = 4t,

B = $(4t-1)\left[\frac{(8t-3)(8t-4)}{3}-2\right]$, Q_1 = $16t^2$-10t,

Q_2 = $16t^2$ - 10t , Q_3 = $\frac{(4t-1)(4t-2)(4t-3)}{6}$,

R = (4t - 1)($32t^2$ – 28t + 3), K = 6t, Λ = (4t - 2)[$48t^2$ - 34t + 7] – (18t - 5).

Corollary 2.5 : The existence of a BIB design with parameters V = 2t-1, b = 4t-2, r = 2t-2, k = t-1, λ = t-2, implies the existence of a BQD with parameters, V = 2t,

B = (4t-2) $\left[\frac{(8t-5)(8t-6)}{3}-2\right]$,

$Q_1 = (4t-2)\left[\frac{(4t-2)(4t-3)}{2}-1\right]$,

$Q_2 = (4t-2)\left[\frac{(4t-2)(4t-3)}{2}-1\right]$,

$Q_3 = \frac{(4t-2)(4t-3)(4t-4)}{6}$, R = (4t-2)($32t^2$ – 44t + 12), K = 2t, Λ = 2(4t-3) [(t-1)(8t-7) + $(4t-2)^2$] – (18t-10).

Example 2.2: Let us consider BIB design with parameters V = b = 3, r = k = 1, λ = 0. It implies self complementary BIB design with parameters V = 4, $b' = 6,\ r' = 3,\ k' = 2,\ \lambda' = 1$. On applying Theorem 2.2, it is developed as BQD (Table 2) after deleting the repeated blocks.

Table 2: The number of blocks in BQD with parameters V=4, B=14, Q_1=6, Q_2=6, Q_3=1, R=21, K=6, Λ=29

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}
1	1	1	1	1	1	1	1	1	2	1	1	1	1
2	2	1	1	2	1	1	1	1	2	2	2	2	1
3	2	2	2	3	3	2	2	3	3	2	2	3	2
4	3	3	2	3	3	3	2	2	3	3	2	3	2
4	4	4	4	4	4	3	3	3	4	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4	3

Note: Efficiency of this design is-0.920.

Theorem 2.3: The addition of two BIB designs with parameters V = 2k, b_1 = 2r, r_1, k_1, λ_1 and v, b_2, r_2, k+1, λ_2 will provide the following existence of a BQD with following parameters

V = 2k+1, $B=\binom{b_1+b_2}{3}$, $Q_1=\binom{r_1+r_2}{1}\times$ $\binom{b_1+b_2-r_1-r_2}{2}$, Q_2 = $\binom{r_1+r_2}{2}\times\binom{b_1+b_2-r_1-r_2}{1}$,

Q_3 = $\binom{r_1+r_2}{3}$, K = 3(k + 1), $\Lambda=9\binom{\lambda_1+\lambda_2}{3}+$

$$12\left[\binom{\lambda_1+\lambda_2}{2}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\right]+$$

$$6\left[\binom{\lambda_1+\lambda_2}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{2}\right]+$$

$$4\left[\binom{\lambda_1+\lambda_2}{2}\binom{b_1+b_2-2(r_1+r_2)+(\lambda_1+\lambda_2)}{1}+\right.$$

$$\left.\binom{\lambda_1+\lambda_2}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\right]+4$$

$$\left[\binom{\lambda_1+\lambda_2}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\right.$$

$$\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{1}+$$

$$\binom{r_1+r_2-(\lambda_1+\lambda_2)}{2}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}+$$

$$\binom{\lambda_1+\lambda_2}{1}\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{2}+$$

$$\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}$$

$$\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{1}$$

Proof: BQD are constructed with the addition of two BIB designs in which first BIB consists of the condition that V = 2k, b = 2r. Hence the number of treatments in 2k+1, and the number of blocks is B = $\binom{b_1+b_2}{3}$. The other parameters are explained below:

Q_1 : Total number of replications is $r_1 + r_2$. Hence

$Q_1 = \binom{r_1+r_2}{1}\times\binom{b_1+b_2-r_1-r_2}{2}$

$Q_2 = \binom{r_1+r_2}{2}\times\binom{b_1+b_2-r_1-r_2}{1}$.

$Q_3 = \binom{r_1+r_2}{3}$

R : Replication number R for treatment x is $R = Q_1 + 2Q_2 + 3Q_3$.

Hence $R = \binom{r_1+r_2}{1}\times\binom{b_1+b_2-r_1-r_2}{2}+2\binom{r_1+r_2}{2}\times$

$\binom{b_1+b_2-r_1-r_2}{1}+3\binom{r_1+r_2}{3}$.

K : 3(k + 1)

Λ : This parameter will consist of (3,3), (3,2), (2,3), (3,1), (1,3), (2,2), (2,1), (1,2), (1,1) ordered pairs of treatments.

For ordered pair (3,3), we consider 3's of the total λ's. Therefore , it is equal to $\binom{\lambda_1+\lambda_2}{3}$.

For ordered pair (3,2) and (2,3), we consider the 2's of λ's and one of (r-λ) which is equal to $\binom{\lambda_1+\lambda_2}{2}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}$.

For ordered pair (3,1) and (1,3), we consider the 1's of λ's and 2's of (r-λ) which is equal to $\binom{\lambda_1+\lambda_2}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}$.

For ordered pair (2,2),we consider 2's of λ's and one of (0,0) and combination of (1,1,0) and (0,1,1) , we consider one of λ's and 2's of (r-λ) which is equal to

$$\binom{\lambda_1+\lambda_2}{2}\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{1}+$$

$$\binom{\lambda_1+\lambda_2}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}$$

For ordered pair (2,1) and (1,2), we consider the one of λ's and one of the (r-λ) and (b-2r+λ) and other combination of (1,0,0) and (0,1,0), we consider 2's and one of (r-λ) which is equal to

$$\binom{\lambda_1+\lambda_2}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}$$

$$\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{1}+$$

$$\binom{r_1+r_2-(\lambda_1+\lambda_2)}{2}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}.$$

For ordered pair (1,1), we consider one of λ's and 2's of (b-2r+λ) and other combination of (1,0,0) and (0,1,0), we consider of 2's of (r-λ) and one of (b-2r+λ) which is equal to

$$\binom{\lambda_1+\lambda_2}{2}\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{2}+$$

$$\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}$$

$$\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{1}$$

Thus,

$$\Lambda = 9\binom{\lambda_1+\lambda_2}{3}+12\left[\binom{\lambda_1+\lambda_2}{2}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\right]+$$

$$6\left[\binom{\lambda_1+\lambda_2}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{2}\right]+4$$

$$\left[\binom{\lambda_1+\lambda_2}{2}\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{1}+\right.$$

$$\left.\binom{\lambda_1+\lambda_2}{1}\binom{(r_1+r_2)-(\lambda_1+\lambda_2)}{1}\binom{(r_1+r_2)-(\lambda_1+\lambda_2)}{1}\right]+4$$

$$\left[\binom{\lambda_1+\lambda_2}{1}\binom{(r_1+r_2)-(\lambda_1+\lambda_2)}{1}\right.$$

$$\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{1}+$$

$$\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}+$$

$$\binom{\lambda_1+\lambda_2}{1}\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{2}+$$

$$\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}\binom{r_1+r_2-(\lambda_1+\lambda_2)}{1}$$

$$\binom{(b_1+b_2)-2(r_1+r_2)+(\lambda_1+\lambda_2)}{1}$$

Hence, the theorem proved.

Example 2.3: Let us consider BIB design with parameters V = 4, b = 6, r = 3, k = 2, λ =1 and BIB designs with parameters V = 4, b = 4, r = 3, K = 3, On applying Theorem 2.3, it is developed as BQD parameters. V = 5, B = 120, Q_1 = 36, Q_2 = 60, Q_3 = 40, R = 276, K = 9, Λ = 363.

Efficiency of this design is -0.730.

Theorem 2.4: The existence of a BIB design with parameters $V' = 2k+1,\ b',\ r',\ k',\ \lambda'$ with $b' = 3r' - 2\lambda'$ implies the existence of balanced quaternary design (BQD) with following parameters V′= 2k′+1, B = b′(b′-1)(b′-2)/6, Q_1 = r′(b′-r′)(b′-r′-1)/2, Q_2 = r′(r′-1)(b′-r′)/2, Q_3 = r′(r′-1)(r′-2)/6, R = r′b′²-3b′r′+2r′/2, K = 3k′, Λ = 2λ′²+ 6λ′ – 4b′λ′ + 28r′²λ′ – 4r′² + λ′b′² – b′λ′ + 2r′²b′/2.

Proof : Hadayat and Wallis (1978) pointed out the use of hadamard matrics the construction of BIB design matrices in the form of N_1,N_2,N_3,N_4. Here. BQD are constructed by taking the combination of three blocks of the N_3.

Hence, $\binom{b'}{3} = \frac{b'(b'-1)(b'-2)}{6}$. Therefore, the total number of blocks is B = $\frac{b'(b'-1)(b'-2)}{6}$.

The parameters V, B, and *K* need no explanation.

Remaining parameters are explained below:

Q_1 : Let us consider a block containing a particular treatment *x*. This will occur by taking the combination of *r* and *b-r* which is equal to

$$Q_1 = \binom{r'}{1}\times\binom{b'-r'}{2} = \frac{r'(b'-r)(b'-r'-1)}{2}$$

Q_2 : This will occur by taking the combination ot two treatments i.e. *r* and *b-r* which is equal to

$$Q_2 = \binom{r'}{2}\times\binom{b'-r'}{1} = \frac{r'(r'-r)(b'-r')}{2}$$

$$Q_3 = \binom{r'}{3} = \frac{r'(r'-1)(r'-2)}{6}$$

R : Replication number *R* for treatment *x* is *R* = Q_1 + $2Q_2$ + $3Q_3$.

Hence, R = r′b′²-3b′r′+2r′/2

K : 3k′

Λ : This parameter will consist of (3,3), (3,2), (2,3), (3,1), (1,3), (2,2), (2,1), (1,2), (1,1) ordered pairs of treatments.

For ordered pair (3,3), we consider 3’s of the total λ’s.Therefore , it is equal to $\binom{\lambda'}{3}$.

For ordered pair (3,2) and (2,3), we consider the 2’s of λ’s and one of (r-λ) which is equal to $\binom{\lambda'}{2}\times\binom{r'-\lambda'}{1}$

For ordered pair (3,1) and (1,3), we consider the 1’s of λ’s and 2’s of (r-λ) which is equal to $\binom{\lambda'}{1}\times\binom{r'-\lambda'}{2}$

For ordered pair (2,2),we consider 2’s of λ’s and one of (0,0) and combination of (1,1,0) and (0,1,1) , we consider one of λ’s and 2’s of (r-λ) which is equal to

$$\binom{\lambda'}{2}\binom{b'-2r'+\lambda'}{1}+\binom{\lambda'}{1}\binom{r'-\lambda'}{1}\binom{r'-\lambda'}{1}$$

For ordered pair (2,1) and (1,2), we consider the one of λ’s and one of the (r-λ) and (b-2r+λ) and other combination of (1,0,0) and (0,1,0), we consider 2’s and one of (r-λ) which is equal to

$$\binom{\lambda'}{1}\binom{r'-\lambda'}{1}\binom{b'-2r'+\lambda'}{1}+\binom{r'-\lambda'}{2}\binom{r'-\lambda'}{1}$$

For ordered pair (1,1), we consider one of λ’s and 2’s of (b-2r+λ) and other combination of (1,0,0) and (0,1,0), we consider of 2’s of (r-λ) and one of (b-2r+λ) which is equal to

$$\binom{\lambda'}{1}\binom{b'-2r'+\lambda'}{2}+\binom{r'-\lambda'}{1}\binom{r'-\lambda'}{1}\binom{b'-2r'+\lambda'}{1}$$

Thus, $\Lambda = 9\binom{\lambda'}{3}+12\left[\binom{\lambda'}{2}\binom{r'-\lambda'}{1}\right]+$

$$6\left[\binom{\lambda'}{1}\binom{r'-\lambda'}{2}\right]+$$

$$4\left[\binom{\lambda'}{2}\binom{b'-2r'+\lambda'}{1}+\binom{\lambda'}{1}\binom{r'-\lambda'}{1}\binom{r'-\lambda'}{1}\right]+$$

$$4\left[\binom{\lambda'}{1}\binom{r'-\lambda'}{1}\binom{b'-2r'+\lambda'}{1}+\binom{r'-\lambda'}{2}\binom{r'-\lambda'}{1}\right]+$$

$$\binom{\lambda'}{1}\binom{b'-2r'+\lambda'}{2}+\binom{r'-\lambda'}{1}\binom{r'-\lambda'}{1}\binom{b'-2r'+\lambda'}{1}$$

$\Lambda = 2\lambda'^2 + 6\lambda' - 4b'\lambda' + 28r'^2\lambda' - 4r'^2 + \lambda' b'^2 - b'\lambda' + 2r'^2b'/2$.

Hence Q.E.D.

Corollary 2.6 : The existence of a BIB design with parameters $V = b = 4t-1$, $r = k = 2t - 1$, $\lambda = t - 1$,implies the existence of a BQD with parameters, $V = (4t-1)$, $B = 32t^3 - 48t^2 + 22t - 3/3$, $Q_1 = t(2t-1)^2$, $Q_2 = 2(2t^3-3t^2 + t)$, $Q_3 = 4t^3-12t^2 + 11t-3/3$, $R = (2t-1)(16t^2 - 20t +6)/2$, $K = 3(2t-1)$, $\Lambda = (t-1)(-112t^2 - 110t-23) - (2t - 1)^2 (-4t+5)/2$.

Corollary 2.7: The existence of a BIB design with parameters $V = 2t-1, b = 4t-2, r = 2t-2, k = t-1, \lambda = t-2$, implies the existence of a BQD with parameters, $V = 2t$, $B = 4(2t-1)(4t-3)(t-1)/3$, $Q_1 = 2t(t-1)(2t-1)$, $Q_2 = 2t(t-1)(2t-3)$, $Q_3 = 2(t-1)(2t-3)(t-2)/3$, $R = (t-1)(16t^2 - 28t + 12)$, $K = 3(t-1)$, $\Lambda = (t-2)(128t^2 - 256t+ 128)-(2t-2)^2(8t)/2$.

Example 2.4: Let us consider BIB design with parameters $V = b = 3$, $r = k = 1$, $\lambda = 0$. By combining itself design having parameters $V = 3$, $b = 6$, $r = 2$, $k = 1$, $\lambda = 0$. On applying Theorem 2.4, it is developed as BQD (Table 3).

Table 3: The number of blocks in BQD with parameters $V=3, B=20, Q_1=12, Q_2=4, Q_3=0, R=20, K=3, \Lambda=16$

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}	B_{19}	B_{20}
1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	1
2	2	2	2	3	3	3	1	1	2	3	3	3	1	1	2	1	1	2	2
3	1	2	3	1	2	3	2	3	3	1	2	3	2	3	3	2	3	3	3

Note: Efficiency of this design is- 0.80.

3. Applications

Deleting eight blocks given in Example 2.4, it can be used for conducting intercropping experiments in two replicates when the intercrops are subdivided into various groups based on agronomic practices. We construct designs for experiments where each plot consists of one main crop and six intercrops in such a way that each of these intercrops is selected from a group of intercrops following Rao and Rao (2001).

Now, let us consider an intercropping experiments using one main crop and six intercrops where the intercrops are divided into three groups S_1,S_2,S_3 with two in each group *viz*.,S_1 = [1,2], S_2 =[3,4], S_3 = [5,6] .Let us designate the symbols 0, 2 of first row of BQD with intercrops 1, 2 of S_1 , second row with intercrops 3, 4 of S_2, third row with intercrops 5,6 of S_3. Taking into the consideration the column of the array as the plots of the intercropping experiments in addition to one main crop in each plot with two replicates the resulting intercropping experiments will consist of the following twelve plots on the basis of the blocks given in the **Example 2.4.**

(m,2,5); (m,4,5); (m,2,3); (m_1,3,6); (m,2,5); (m,2,3); (m,1,4); (m,1,6); (m,4,5); (m,1,4); (m,3,6); (m,1,6).

This layout of the intercropping experiment is found to be superior having one main crop and six intercrops in a six plots rather than that of Sharma *et al.* (2013).

In the context of an actual example of intercropping experiment, Takim (2012) have used the different mix-proportions and planting patterns of maize (*Zea mays* L.) and cowpea (*Vigna unguiculata* L.) for the comparisons of sole cropping of each crop during 2010 and 2011 growing seasons under the southern Guinea savanna conditions in Nigeria. The experiment comprised of 6 treatments: sole maize (51,282 plants ha^{-1}), sole cowpea (61,538 plants ha^{-1}) and 4 maize – cowpea intercropping mix-proportion: 100 maize: 100 cowpea, 50 maize: 50 cowpea ,60 maize:40 cowpea and 40 maize:60 cowpea using randomized complete block design with three replications. Evaluation of the intercropping patterns was performed on basis of several intercropping indices.The study revealed that the mix-proportion of 50 maize:50 cowpea gave a similar grain yield compared to other intercropped plots.The study also revealed that intercropping systems could be an eco-friendly approach for reducing weed problems through non-chemical methods, mix-proportion of 50 maize:50. cowpea planted on alternate rows could be a better intercropping pattern.

In another example of intercropping experiment, Pandey *et al.* (2003) have studied the effect of maize (*Zea mays* L.) based intercropping system on maize yield as main crop and six intercrops *viz.,* pigeonpea, sesamum, groundnut, blackgram,turmeric and forage *meth* by conducting an experiment during the rainy seasons of 1998 and 1999 at the research farm of Rajendra Agricultural University, Pusa, Samastipur(Bihar). The experiment consisted of six intercrops with one main crops was conducted in randomized complete block design with four replications. Maize was grown at a spacing of 75 cm. Row spacing in sole as well as in intercropping on 26 and 22 June respectively in the first and second year of experimentation. accommodated between two rows of maize. The intra row spacing of 30, 30, 10, 15, 10, and 15 cm were maintained by thinning for six intercrops.One row of pigeon pea at a distance of 75 cm and 2 rows of other intercrops at 30 cm distance were accommodated between two rows of maize. The intra row spacing of 30, 30, 10, 15, 10, and 15 cm were maintained by thinning for six intercrops.

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